

TREATMENT OF THE QCD COUPLING IN HIGH ENERGY PROCESSES

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The treatment of the running QCD coupling in evolution equations is discussed. It is shown that the use of the virtuality of ladder (vertical) partons as the scale for QCD coupling in every rung of ladder graphs is an approximation that holds for DIS at large x only. On the contrary, in the small x region the coupling depends on the virtuality of s -channel (horizontal) gluons. This observation leads to different results for the Regge-like processes and DIS structure functions at small x .

I. INTRODUCTION

In perturbative QCD, calculations of physical quantities such as scattering amplitudes or cross sections beyond the Born approximation basically requires one to account for running α_s effects. In fixed-order calculations, α_s becomes running in a straightforward way, by direct calculation of Feynman graphs. However, in various approaches, scattering amplitudes or cross sections are often calculated to all orders of the perturbation theory. Generally this means that some equations are constructed and solved and when such equations include the running of α_s , the argument of α_s is usually fixed by certain prescriptions which are based on different approximations. For example, for hard QCD processes exploiting DGLAP evolution [1], one takes

$$\alpha_s = \alpha_s(k_\perp^2) \quad (1)$$

in every rung of quark/gluon ladder, with k_\perp being the transverse momentum of the ladder parton. As a consequence, in the evolution equations

$$\alpha_s = \alpha_s(Q^2), \quad (2)$$

with $\sqrt{Q^2}$ being the upper limit for integration over transverse momenta k_\perp of ladder partons. In particular, when the DGLAP is used for calculation of the structure functions of the Deep Inelastic Scattering (DIS), $-Q^2$ is the virtuality of the photon. The dependence of α_s given by Eqs (1,2) follows from the results of ref. [2] for the treatment of α_s in hard QCD processes. On the other hand, some important high energy QCD processes are Regge-like, rather than hard. In particular, high energy forward and backward scattering and DIS at small x are typical Regge-like processes. The use of the hard -like prescription [2] of the QCD coupling for these Regge processes should have been justified somehow. Indeed, much recent work uses the argument of α_s given by Eqs. (1, 2) without any comments. For example, in ref [3] Eq. (1) is used for incorporating running α_s into the BFKL equation though it is well-known that the BFKL is constructed especially for the Regge kinematics. Also, works [4] suggest $\alpha_s = \alpha_s(Q^2)$ for the polarised DIS structure function g_1 at small x . On the other hand, instead of exploiting directly Eq. (1) refs [5], [6] prescribe a modified parametrisation of α_s , namely

$$\alpha_s = \alpha_s(k_\perp^2/\beta), \quad (3)$$

where β is the parameter of the standard Sudakov representation of the ladder momentum k ¹:

$$k_\mu = \alpha q_\mu + \beta p_\mu + k_{\perp\mu}. \quad (4)$$

Contrary to refs. [1] - [6], we have suggested [7] a quite different treatment of α_s in the evolution equations for the non-singlet structure functions at small x . It is interesting that when the argument of α_s is discussed in ref [8], the first assumptions about the argument look similar to Eqs. (1) though final result agrees rather with [7] than with

¹momenta p and q must obey $2pq \gg p^2 \sim q^2 \sim 0$, otherwise they are arbitrary

Eq. (1). In the present work we extend the treatment of α_s suggested in [7] for the non-singlet structure functions, which are based on a quark ladder, to the more complicated case of gluon ladders, forming the base for the singlet structure functions of DIS. We show that the treatment of α_s suggested in refs. [2] for hard processes and often used subsequently for description of different high energy QCD processes is an approximation valid only for hard QCD processes. This treatment cannot be used for the Regge kinematics. We show that the α_s -dependence given by Eq. (1) is valid only for hard QCD processes where values of all Mandelstam variables are of the same order. In particular, for DIS it means that $\alpha_s = \alpha_s(Q^2)$ (or $\alpha_s(k^2)$) in evolution equations only when $x \sim 1$. On the contrary, the treatment of α_s of Ref. [7] is valid for the evolution of DIS structure functions at small x and for other high energy QCD processes in the Regge kinematics. The paper is organised as follows: in sect 2 we discuss the treatment of α_s in elementary parton processes. In sect. 3 we extend it to partonic ladders in the Regge kinematics and compare our results with results those of ref. [2]. Finally, sect. 4 contains our concluding remarks.

II. INCORPORATING RUNNING α_s INTO THE BORN SCATTERING AMPLITUDES

In this section we discuss the argument of α_s in elementary parton processes. In order to avoid unnecessary complications we regard all these partons as slightly off-shell. We specify this fact explicitly in Eq. (14). Let us consider first the scattering amplitude M_{qq} for the process of annihilation of a quark- antiquark pair into a quark -antiquark pair of the same flavour:

$$q(p_1) + \bar{q}(p_2) \rightarrow q(p'_1) + \bar{q}(p'_2) \quad (5)$$

in the forward kinematical region

$$s = (p_1 + p_2)^2 \approx -u = (p'_2 - p_1)^2 \gg -t = -(p'_1 - p_1)^2 . \quad (6)$$

In the Born approximation M_{qq}^{Born} is given by contributions of three Feynman graphs. In each such graph the intermediate gluon has momentum k , so that either $k^2 = s$ or $k^2 = u$ or at last $k^2 = t$. Such intermediate gluons are often addressed as s , u or t -channel gluons respectively.

According to the Optical theorem

$$\sigma_{tot} \sim s^{-1} \Im_s M|_{t=0} , \quad (7)$$

so only the graph with an intermediate s -channel gluon in M_{qq}^{Born} contributes to the total cross section of the process. As our final goal is cross sections, we consider only this graph and denote its contribution by \tilde{M}_{qq}^{Born} . Adding the part of radiative corrections to \tilde{M}_{qq}^{Born} which makes α_s running \tilde{M}_{qq}^{Born} transforms into

$$\tilde{M}_{qq} = -4\pi\alpha_s(s) \frac{\bar{u}(p_2)\gamma_\mu u(p_1)\bar{u}(p_2)\gamma_\mu u(p_1)}{s + i\epsilon} . \quad (8)$$

where α_s depends on s . In Eq. (8), we have dropped the factor 4π and the colour structure. The Feynman graphs corresponding to \tilde{M}_{qq} are depicted in Fig. 1. The well-known BLM [9] arguments allow one to consider only quark loop contributions to α_s and it is easy to show that the argument of α_s in Eq. (8) is indeed s . To this end let us note first that at high energies and for space-like q obeying

$$\Lambda^2 \equiv \Lambda_{QCD}^2 \ll -q^2 \quad (9)$$

α_s is given by

$$\alpha_s(q^2) = \frac{1}{b \ln(-q^2/\Lambda^2)} , \quad b = (11N - 2n_f)/(12\pi) . \quad (10)$$

As is well-known, in Eq. (10) the contribution proportional to the number of colours, N comes from graphs with gluon loops whereas the contribution proportional to n_f comes from the QED-like graphs where the gluon propagator is saturated only by quark loops. Obviously, the main, logarithmic contribution of quark loops is proportional to $\ln(s)$. As eventually quark and gluon loop contributions combine into Eq. (10), the argument of α_s in Eq. (8) is indeed s .

Now let us repeat the above reasoning, replacing the quarks by gluons, i.e. let us consider the forward scattering process

$$g(p_1) + g(p_2) \rightarrow g(p'_1) + g(p'_2) \quad (11)$$

in the forward region Eq. (6). We denote the Born amplitude for the forward scattering of gluons by M_{gg}^{Born} and, similarly to Eq. (8), denote the Born approximation for the part of the forward amplitude with s -channel intermediate gluon by \tilde{M}_{gg}^{Born} (see Fig. 2). As is well-known, the Slavnov-Taylor [10] identities state that accounting for radiative corrections to \tilde{M}_{gg}^{Born} lead to the same dependency, $\alpha_s = \alpha_s(s)$. Therefore both for the quark and gluon scattering, adding radiative corrections to the part of the Born scattering amplitude with an s -channel gluon transforms it into

$$\tilde{M} = -4\pi\alpha_s(s) \frac{R(s)}{s + i\epsilon} \quad (12)$$

where we have dropped the color factors. R includes vertices and spin dependence. For example, for the scattering of on-shell quarks, $R(s) = R_q(s) = 2s$. For the gluon scattering, $R = R_g$ depends on polarisations of gluons. We consider only its part, \tilde{M} , with s -channel gluon, instead of the whole scattering amplitude M , disregarding parts of M with the t and u -channel gluons because only \tilde{M} has non-zero imaginary part with respect to s . Eq. (12) then leads to

$$\Im_s M \equiv \Im \tilde{M} = -\Im \alpha_s(s) \left(\frac{4\pi R(s)}{s} \right) + \Re \alpha_s(s) \pi \delta(s) (4\pi R(s)) \quad (13)$$

where we have used $\Im R(s) = 0$. We remind the reader that throughout this paper we assume all partons to be slightly off-shell. For the on-shell quarks, the second term in the rhs of Eq. (13) is zero. It is easy to see that $\alpha_s(s)$, given by Eq. (10) when its argument is space-like, can be rewritten as

$$\alpha_s(s) = \frac{1}{b \ln(-s/\Lambda^2)} = \frac{\ln(s/\Lambda^2) + i\pi}{b[\ln^2(s/\Lambda^2) + \pi^2]}, \quad (14)$$

when s is time-like. Indeed, any scattering amplitude $M(s)$ has a non-zero imaginary part in s when s is positive. In particular when M depends on s through logarithms, it means that

$$M = M(\ln(-s)) \quad (15)$$

when s is positive. However, Eq. (15) does not fix the phase of $-s = se^{i\phi}$. In order to fix ϕ , one can use the fact that M does not have an imaginary part when s is negative. Then, the analytical continuation of M from positive s to negative s is defined through the upper path, because the cut in the s -plane when $s > 0$. In doing so, s acquires the factor $e^{i\pi}$ leading to $\phi = -\pi$. Thus, instead of Eq. (15) one has

$$M = M(\ln(s) - i\pi) \quad (16)$$

when s is positive. Combining now Eq. (13) and Eq. (14), we obtain

$$\Im \tilde{M} = \frac{4\pi^2 R(s)}{b[\ln^2(s/\Lambda^2) + \pi^2]} \left[\ln(s/\Lambda^2) \delta(s) - \frac{1}{s} \right]. \quad (17)$$

Taking $\Im_s M$ corresponds to cutting all intermediate states in the s -channel and summing up such contributions. The first term in the squared brackets in Eq. (17) corresponds to the case when the s -channel intermediate state is an on-shell gluon whereas the second term corresponds to many on-shell partons in the intermediate state. Using the δ -function in Eq. (17) when integrating the first term over s leads to a result which is singular in the infrared region. The reason for that is obvious: one can apply Eq. (10) for $\alpha_s(s)$ only when $s \gg \Lambda^2$. In order to fix this we replace $\delta(s)$ by $\delta(s - \mu^2)$ in Eq. (12), with the infrared cut-off $\mu \gg \Lambda$, arriving therefore at the expression

$$\Im \tilde{M} = \frac{4\pi^2 R(s)}{b[\ln^2(s/\Lambda^2) + \pi^2]} \left[\ln(s/\Lambda^2) \delta(s - \mu^2) - \frac{1}{s} \right] \quad (18)$$

for s -channel imaginary part of M_{qq} and M_{gg} .

Finally let us consider the quark-gluon scattering, with both gluons off-shell, $k^2 = (k')^2 < 0$:

$$q(p) + g(k) \rightarrow q(p') + g(k') \quad (19)$$

in the forward region (6). In the Born approximation the only Feynman graph with non-zero imaginary part in s is depicted in Fig. 3. Incorporating the radiative corrections to this graph in the same way as was done for quark-quark and gluon-gluon scattering and using the BLM approach [9], we conclude that for qg - scattering

$$\alpha_s = \alpha_s(k^2), \quad (20)$$

which agrees with the treatment of α_s suggested in ref. [2]. Obviously, $\Im_s \alpha_s(k^2) = 0$ and therefore

$$\Im \tilde{M}_{gg} = \frac{4\pi^2 R(s)}{b \ln(s/\Lambda^2)} \delta(s - \mu^2) \quad (21)$$

instead of Eq. (18).

III. QCD COUPLINGS IN LADDER GRAPHS

Now let us allow all partons in Eqs. (5),(11) and (19) to be off-shell so that their virtualities cannot be neglected. It is obvious that letting the on-shell partons which were considered in sect. 2 be off-shell does not change our conclusions concerning the argument of α_s in these scattering processes. Our formulae Eqs. (18),(21) also remain valid for off-shell parton scattering, save a change of R . However, letting the partons be off-shell converts the amplitudes we have considered into the rungs of partonic ladder graphs. Therefore we conclude that incorporating the running QCD coupling into the gluon-gluon and quark-quark rungs fixes the time-like virtuality of the intermediate s -channel gluon (the horizontal gluon line in Figs. 1-4) as the scale of α_s . In contrast to this, the scale of α_s for the quark-gluon and gluon-quark rungs is the space-like virtuality of the vertical gluon lines.

Therefore the integrand of the quark ladder depicted in Fig. 4 contains $n + 1$ QCD couplings in such a way:

$$\alpha_s((p_1 - k_1)^2) \alpha_s((k_1 - k_2)^2) \alpha_s((k_2 - k_3)^2) \dots \alpha_s((p_2 + k_n)^2). \quad (22)$$

When quark lines in Fig. 4 are replaced by the gluon lines, the integrand of such purely gluon ladder (we keep the same notations for the gluon ladder momenta as for the quark ones) contains the same succession of QCD couplings with the same arguments as Eq. (22). However, some interesting objects (e.g. for the singlet DIS structure function g_1) are made of mixing of the quark and gluon rungs as shown in Fig. 5, so that the integrand corresponding to Fig. 5 is proportional to

$$\alpha_s(k_1^2) \alpha_s((k'_1)^2) \alpha_s((k_3 - k_4)^2) \dots \alpha_s(k_n^2) \alpha_s^2((k'_n)^2) \quad (23)$$

instead of the DGLAP succession²

$$\alpha_s(k_1^2) \alpha_s(k_2^2) \alpha_s(k_3^2) \dots \alpha_s^2(k_n^2). \quad (24)$$

A further difference concerning treatment of α_s arises when those ladders are used for calculation of cross sections (e.g. DIS structure functions), which contain the imaginary part, $\Im_s A$ of these ladders: as all s -channel gluons (the horizontal lines in Fig. 5) are time-like, the related α_s have non-zero $\Im_s \alpha_s$ too. Accounting for them makes the pattern much more complicated than the standard DGLAP approach.

Ladder graphs incorporating all orders in α_s are essential ingredient of the DIS structure functions. We demonstrate now how and when it is possible to justify the DGLAP-like argument (1) of α_s for them. To this aim let us suppose that some DIS structure function Φ obeys a Bethe-Salpeter equation. The part of this equation involving the s -channel cut of the ladder virtual gluon is represented by the graph in Fig. 6. For example, it corresponds to the DGLAP equation or (after replacing the photon lines in Fig. 6 by gluon ones) - to the first term of the BFKL equation with running α_s . After simplification of the spin structure, its contribution, $\Phi(s, Q^2)$ can be written as follows :

$$\Phi(s, Q^2) = (p + q)^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\Phi((q + k)^2, Q^2, k^2)}{(k^2)^2} \frac{4k_\perp^2}{(q + k)^2} \Im_s 4\pi \frac{\alpha_s((p - k)^2)}{(p - k)^2} \quad (25)$$

where we have used the standard notations: q stands for the virtual photon momentum, $Q^2 = -q^2 > 0$ and k_\perp is transverse to the plane formed by q and p . The intermediate partons with momenta k in Fig. 6 can be either quarks

²for the ladder parton momenta $k_i^2 \approx -k_{i\perp}^2$

or gluons. The external parton with momentum p is assumed to be on-shell and we will neglect its virtuality, which is not important for our conclusions. As we are discussing the treatment of α_s , we have dropped unessential numerical factors from rhs of Eq. (25) and have used the same notation Φ both for the on-shell structure function in the lhs of Eq. (25) and for the off-shell one in the rhs. Φ is supposed to be a logarithmic function of s . In contrast to the conventional DGLAP or BFKL cases, the rhs of Eq. (25) contains $\Im\alpha_s$ instead of α_s . Also, the argument of α_s is the virtuality of the horizontal gluon. However under certain conditions, one can rewrite Eq. (25) in the form containing α_s with the DGLAP argument k_\perp^2 . To show this let us use the Sudakov representation for k

$$k_\mu = \alpha(q + xp)_\mu + \beta p_\mu + k_{\perp\mu} . \quad (26)$$

where $x = Q^2/s$, $s \equiv 2pq$, and then introduce the virtuality $m^2 = (p - k)^2$ of the s -channel gluon as a new variable instead of α . As $k^2 = -(\beta m^2 + k_\perp^2)/(1 - \beta)$, $(q + k)^2 = [(\beta - x)[s(1 - \beta) - m^2] - k_\perp^2(1 - x)]/(1 - \beta)$, we arrive at

$$\begin{aligned} \Phi(s, Q^2) &= \frac{1}{\pi^2} \int dk_\perp^2 d\beta dm^2 \frac{k_\perp^2}{[\beta m^2 + k_\perp^2]^2} \Phi\left(\frac{[(\beta - x)[s(1 - \beta) - m^2] - k_\perp^2(1 - x)]}{(1 - \beta)}\right. \\ &\quad \left. Q^2, \frac{(\beta m^2 + k_\perp^2)}{(1 - \beta)}\right) \frac{s(1 - \beta)^2}{[(\beta - x)[s(1 - \beta) - m^2] - k_\perp^2(1 - x)]} \Im_s \frac{\alpha_s(m^2)}{m^2} . \end{aligned} \quad (27)$$

The integration over β is supposed to yield logarithmic contributions. It is necessary for BFKL and important also for DGLAP when applied in the small- x region. To this accuracy the integration over β in Eq. (27) can be written as $d\beta/\beta$, the main contribution coming from the region

$$1 \gg \beta \geq \beta_{min} , \quad (28)$$

with

$$\beta_{min} = x + \frac{k_\perp^2(1 - x)}{s - m^2} \quad (29)$$

as the lowest limit of the integration. Therefore Eq. (27) becomes

$$\begin{aligned} \Phi &\approx \frac{1}{\pi^2} \int_0^\infty d^2 k_\perp \int_{\beta_{min}}^1 \frac{d\beta}{\beta} \int_0^\infty dm^2 \Phi\left(\beta(s - m^2) - k_\perp^2(1 - x), Q^2, (\beta m^2 + k_\perp^2)\right) \\ &\quad \frac{s}{(s - m^2)} \frac{k_\perp^2}{[\beta m^2 + k_\perp^2]^2} \Im\left(\frac{\alpha_s(m^2)}{m^2}\right) . \end{aligned} \quad (30)$$

When the dependence of β_{min} and Φ upon m^2 in the rhs of Eq. (30) can be neglected, one can interpret the integration over m^2 as a dispersion integral for α_s :

$$\begin{aligned} \frac{1}{\pi} \int_0^\infty dm^2 \frac{k_\perp^2}{[\beta m^2 + k_\perp^2]^2} \Im\left(\frac{\alpha_s(m^2)}{m^2}\right) &\approx \\ \frac{1}{\pi} \int_0^\infty dm^2 \frac{1}{[\beta m^2 + k_\perp^2]} \Im\left(\frac{\alpha_s(m^2)}{m^2}\right) &\approx \frac{\alpha_s(-k_\perp^2/\beta)}{k_\perp^2} . \end{aligned} \quad (31)$$

leading to an explicit α_s dependence. In particular this is possible when Φ in the integrand of Eq. (30) does not depend on k^2 and when x is big enough to satisfy

$$\beta_{min} \approx x \gg k_\perp^2(1 - x)/s . \quad (32)$$

Therefore the argument m^2 of α_s in this kinematical condition is

$$-k_\perp^2/x \leq m^2 = -k_\perp^2/\beta \leq -k_\perp^2 \quad (33)$$

Then the integration over β is in the region $x \leq \beta \ll 1$ and Eq. (32) is equivalent to the condition $k_\perp^2(1 - x) < sx = Q^2$ which is exactly the DGLAP region. According to Eq. (32), as x must not be small, the dependence $\alpha_s \approx \alpha_s(k_\perp^2)$ of Eq. (33) actually coincides with the parametrisation suggested in Ref [2].

On the other hand, when $x \rightarrow 0$, β_{min} depends on m^2 :

$$\beta_{min} \approx \frac{k_\perp^2}{s - m^2} \quad (34)$$

and, as well known, Φ in the rhs of Eq. (30) depends on k^2 . This makes impossible the use of Eq. (31) for performing the integration over m^2 . However, to the leading logarithmic accuracy, when $m^2 \ll s$ and $|k^2| \approx k_\perp^2 \gg \beta m^2$, one can still perform the integration in Eq. (31), without using the dispersion relation, as:

$$\begin{aligned} \frac{1}{\pi} \int_0^\infty dm^2 \frac{k_\perp^2}{[\beta m^2 + k_\perp^2]^2} \Im\left(\frac{\alpha_s(m^2)}{m^2}\right) &\approx \frac{1}{k_\perp^2} R, \\ R \equiv \frac{1}{\pi} \int_{\Lambda^2}^{k_\perp^2/\beta} \frac{dm^2}{m^2} \Im\alpha_s(m^2) &= \frac{1}{b} \int_0^{\ln(k_\perp^2/\beta\Lambda^2)} \frac{dz}{z^2 + \pi^2} \end{aligned} \quad (35)$$

where we have used Eq. (14) for $\Im\alpha_s$. Let us note that, contrary to Eq. (31), the argument of α_s in Eq. (35) is time-like. Introducing the infrared cutoff μ^2 we arrive at the result

$$\Phi \approx \frac{1}{\pi} \int_{\mu^2/s}^1 \frac{d\beta}{\beta} \int_{\mu^2}^{s\beta} \frac{dk_\perp^2}{k_\perp^2} \Phi(\beta, Q^2, k_\perp^2) R(k_\perp^2/\beta) \quad (36)$$

for $x \rightarrow 0$, which actually reproduces the first term of the BFKL for the case when the gluon with momentum p is (nearly) on-shell. When π^2 is dropped, the integration in Eq. (35) yields $R = \alpha_s(k_\perp^2/\beta)^3$, otherwise $R = (1/\pi b) \arctan[\ln(k_\perp^2/\beta\Lambda^2)/\pi]$.

Eq. (36) shows that one cannot drop the dependence on β in R . In other words, there is no factorization between the dependence on the longitudinal and transverse momenta at small x .

A simple way to solve such equations (and Eq. (25) in particular) is known from the Regge theory (see e.g. [11]). The point is that the small- x region is actually the Regge kinematical region where one can use the Mellin transform to simplify equations similar to Eq. (25). However, first one should introduce the signature amplitudes $M^{(\pm)} = [M(s, Q^2) \pm M(-s, Q^2)]/2$ so that the structure functions are proportional to $(1/\pi)\Im_s M^{(\pm)}$. Then one can use the Sommerfeld-Watson transform in the asymptotical form:

$$M^{(\pm)}(s/m^2) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{m^2}\right)^\omega \xi^{(\pm)}(\omega) F^{(\pm)}(\omega), \quad (37)$$

where $\xi^{(\pm)} = (e^{-i\pi\omega} \pm 1)/2$ are the signature factors well-known from the Regge theory. Although the transform (37) looks similar to the Mellin transform, actually there is a certain difference between them. Indeed, contrary to the usual Mellin transform, the transform inverse to (37) involves $\Im_s M^{(\pm)}$:

$$F^{(\pm)}(\omega) = \frac{-2}{\sin(\pi\omega)} \int_0^\infty d(s/m^2) (s/m^2)^{-1-\omega} \Im_s M^{(\pm)}(s/m^2). \quad (38)$$

This method can be easily used to solve the Bethe-Salpeter equation (25), with the argument of $\Im\alpha_s$ being time-like. An application of this method has been given in [7] for calculating the nonsinglet structure functions at small x . The fact that one cannot use the parametrisation $\alpha_s = \alpha_s(k_\perp^2)$ for the DIS structure functions at small x suggests in particular that in the generalisation of the BFKL equation, when accounting for running α_s effects, the first term of the new kernel should contain α_s with the time-like argument k_\perp^2/β (at least in the region where k_\perp^2/β dominates over the virtualities of the external gluons) whereas in the second term (corresponding to the virtual contribution) α_s has the space-like argument k_\perp^2 . We will consider the running α_s effects for the BFKL in a forthcoming publication.

IV. CONCLUSION

We have shown that when α_s is running in a general QCD process, providing α_s with the argument k_\perp^2 is not a general rule but an approximation. This approximation is valid for hard processes but fails for the Regge-like ones. It can be used in the DGLAP evolution equations when they are applied in the kinematical region of large x but cannot be used at small x . In this regime we have suggested, for both the quark-quark and the gluon-gluon rungs involving $\Im\alpha_s$, that the argument should be the virtuality of the s -channel (horizontal) gluon. Being incorporated into evolution equations of the Bethe-Salpeter type, this leads to an effective coupling with the argument k_\perp^2/β . When

³A similar parameterization was used in Refs. [5], [6] for the non-singlet structure function.

the virtuality of this gluon is time-like, π^2 -terms appear in expression for $\Im\alpha_s$ (see Eqs. (17),(14)), due to the analytic properties of α_s . These π^2 -terms might be quite important because of their big numerical value. In particular, they have an impact on the values of intercepts of the non-singlet DIS structure functions [7] and are likely to be important for the singlet structure functions as well. However, when the starting point μ_0 of evolution is big enough, π^2 -terms can be easily neglected. It can be used for estimating μ_0 . In particular, we obtained [7] $\mu_0 \approx 5\text{GeV}$ for the non-singlet structure functions.

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Figures

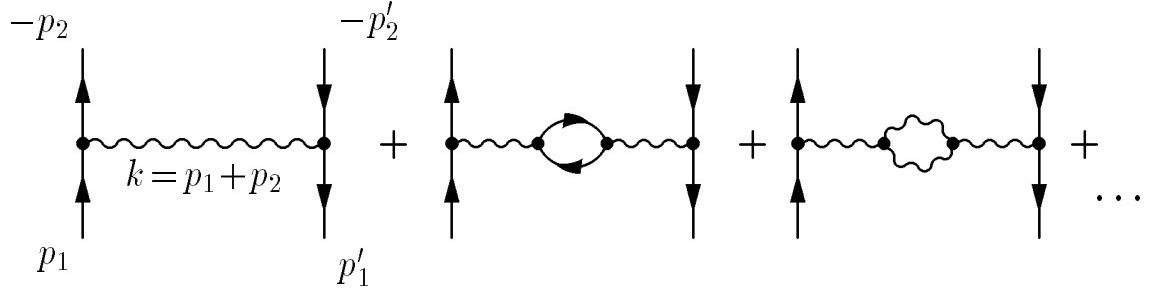


FIG. 1. The Born amplitude \tilde{M}_{qq} , having imaginary part at $s > 0$, with running coupling included.

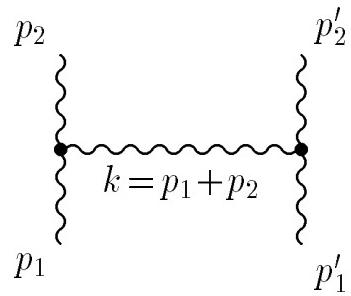


FIG. 2. The Born amplitude \tilde{M}_{gg} , having imaginary part at $s > 0$.

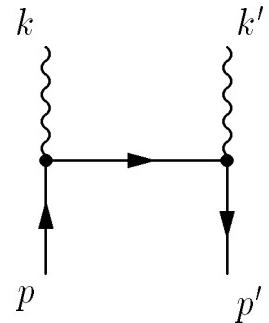


FIG. 3. The Born amplitude \tilde{M}_{qg} , having imaginary part at $s > 0$.

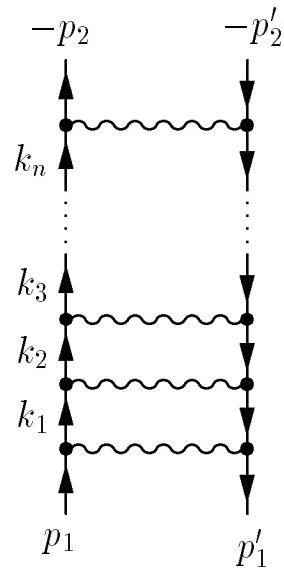


FIG. 4. The quark ladder graph for M_{qq} .

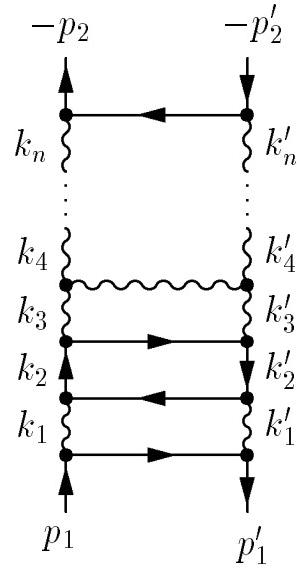


FIG. 5. A general ladder graph for M_{qq} .

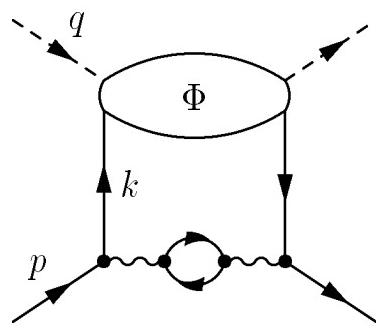


FIG. 6. A graph for DGLAP evolution of a DIS structure function with account of running coupling.